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THERMAL CONDUCTIVITY OF TWO-PHASE AND THREE-PHASE HETEROGENEOUS SOLID MIXTURES

S. C. CHENG and Y. S. LAW
Mechanical Engineering Department, University of Ottawa, Ottawa 2, Canada

and

C. C. Y. KWAN
Physics Department, University of Ottawa, Ottawa 2, Canada

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NOMENCLATURE

k_m, mean thermal conductivity, [Btu/hft °F];
P_d, discontinuous phase volume fraction;
P_{d1}, first discontinuous phase volume fraction;
P_{d2}, second discontinuous phase volume fraction.

INTRODUCTION

THE LINE-SOURCE method was used to measure the thermal conductivity of solid mixtures. The mixtures were made from RTV-60 silicone rubber* as the continuous phase and aluminum, lead, nickel and bismuth† as the discontinuous phase(s). The specifications and properties of the materials are shown in Tables 1 and 2. A comparison is made between the experimental results and those predicted from analytical methods. A discussion of the analytical methods is also presented.

Table 1. Specifications of metal powders used in experiments

Description	Particle dimensions (mean dia., in.)	Purity (%)
Bismuth	0·0058	99·6
Lead	0·0017	99·9
Nickel	0·0017	99·9
Aluminum	0·0037	99·5

EXPERIMENTS

An experimental apparatus for the line-source method was constructed to measure the thermal conductivity of solid mixtures [4]. The design of the apparatus follows closely the device developed by Underwood and McTaggart [5]. Some modifications have been made however to improve the control aspect of the experiment and to provide a more complete collecting of data.

Two samples of single-phase silicone rubber were prepared and thirty-two experiments were conducted. Twelve two-phase samples and six three-phase samples with different fractions of discontinuous phase(s) were used in the experiment.

* Supplier, Canadian General Electrical Company, Toronto, Canada.
† Supplier, Electronic Space Products, Inc., Los Angeles, California, U.S.A.

Table 2. Properties of materials used in experiments

Material	Density (g/cm^3)	Thermal conductivity at 68°F (Btu/hft °F)
Bismuth	9·78 [1]	4·8 [2]
Lead	11·34 [1]	20·0 [2]
Nickel	8·80 [1]	52·0 [2]
Aluminum	2·7 [1]	118 [2]
Rubber (RTV-60)	1·45 [3]	0·222*

* Experimental value from this study.

RESULTS

Single-phase material

A comparison was made between the experimental results and those obtained from the National Bureau of Standards [6]. Since good agreement is observed, the experimental values of silicone rubber obtained are to be used in connection with the following theoretical prediction of the thermal conductivity of mixtures.

Two-phase mixtures

Figure 1 shows typical experimental results of the thermal conductivities versus temperature for two-phase mixtures. Maxwell's equation [7] and Cheng's equation [8] were used to predict the thermal conductivity of these mixtures. A comparison between the experimental data and the values obtained from the theoretical predictions is presented in Table 3.

Three-phase mixtures

Figures 2 and 3 show some experimental results of the thermal conductivities vs. temperature for various three-phase mixtures. Hamilton's equation [9] and Cheng's technique [8] were used for theoretical predictions. In Hamilton's equation, the discontinuous phases were assumed to be spherical in this study. The comparison between the experimental data and the values obtained from the theoretical analysis is presented in Table 4.

Table 3. Comparison of the predicted thermal conductivity with experimental data of two-phase solid mixtures

Experiment		Maxwell's equation		Cheng's equation	
P_d	k_m (Bru/h ft °F)	k_m	Deviation from exp. data (%)	k_m	Deviation from exp. data (%)
Aluminum powder in silicone rubber					
0·05	0·274	0·257	−6·2	0·303	+10·5
0·16	0·442	0·348	−21·3	0·430	−2·7
0·28	0·600	0·479	−20·2	0·619	+3·1
B. Lead powder in silicone rubber					
0·05	0·267	0·256	−4·1	0·295	+10·5
0·16	0·383	0·344	−10·2	0·412	+7·6
0·24	0·497	0·423	−14·9	0·516	+3·8
C. Nickel powder in silicone rubber					
0·05	0·265	0·257	−3·0	0·301	+13·5
0·16	0·379	0·347	−8·5	0·424	+11·8
0·24	0·471	0·429	−8·9	0·537	+14·0
D. Bismuth powder in silicone rubber					
0·05	0·250	0·252	+0·8	0·278	+11·2
0·16	0·341	0·330	−3·2	0·372	+9·1
0·24	0·423	0·398	−5·9	0·451	+6·6

Table 4. Comparison of the predicted thermal conductivities with experimental data of three-phase solid mixtures

P_{d_1}	P_{d_2}	Experiment	Hamilton's equation		Cheng's analysis	
		k_m (Bru/hft °F)	k_m	Deviation from exp. data (%)	k_m	Deviation from exp. data (%)
A. Aluminum and nickel powder in silicone rubber						
0·06	0·04	0·323	0·295	−8·70	0·362	+12·1
0·13	0·09	0·465	0·407	−12·5	0·521	+12·0
B. Aluminum and bismuth powder in silicone rubber						
0·15	0·03	0·445	0·363	−18·4	0·494	+11·0
0·04	0·08	0·368	0·301	−18·2	0·401	+8·9
C. Lead and bismuth powder in silicone rubber						
0·08	0·10	0·400	0·353	−11·8	0·470	+17·5
0·08	0·04	0·342	0·307	−10·2	0·379	+10·8

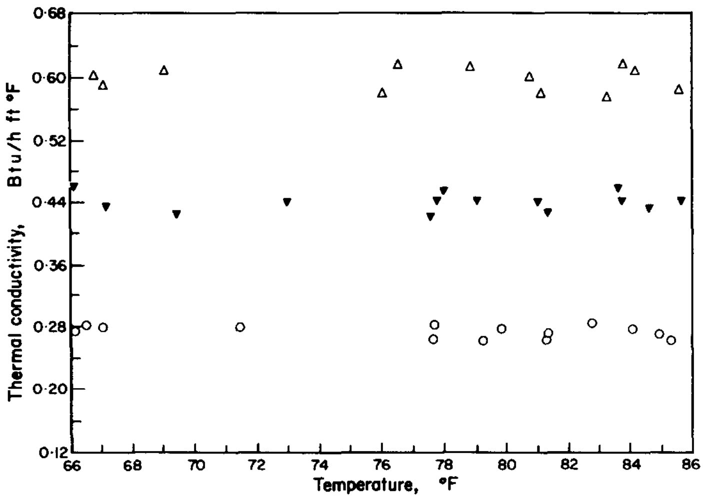

FIG. 1. Thermal conductivity vs. temperature for rubber–aluminum mixtures (measured with current in the range 0·28–0·39 A) △ P_D = 0·28, ▼ P_D = 0·16, ○ P_D = 0·05.

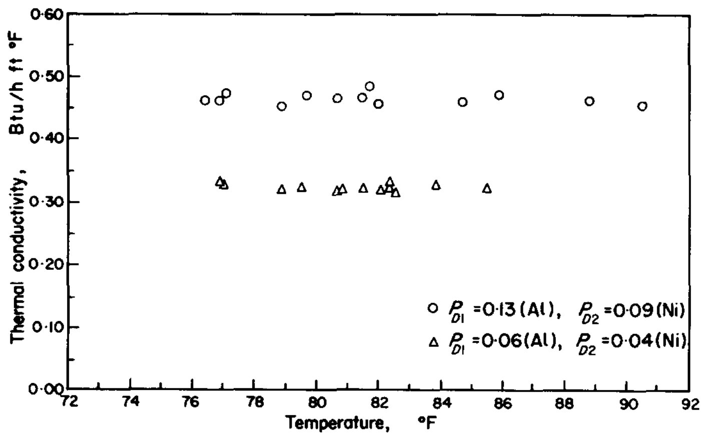

FIG. 2. Thermal conductivity vs. temperature for rubber–aluminum–nickel mixtures.

CONCLUSIONS

For two-phase mixtures, the average percentage of deviation predicted by Maxwell's equation and Cheng's equation are 8·9 per cent and 8·7 per cent, respectively. For three-phase mixtures, the average percentage of deviation predicted by Hamilton's equation and Cheng's analysis is 13·3 per cent and 12·05 per cent, respectively.

Several conclusions can be drawn from this study.

1. The line-source method provides a simple and efficient way for measuring the thermal conductivity of solid mixtures. The experimental results given by the line-source method are consistent and reproducible.

2. Comparing the experimental data of the single-phase material (silicone rubber) with those obtained from the National Bureau of Standards, one can conclude that the line-source method under the condition employed in this study is dependable.

3. In two-phase mixtures, if the volume fraction of the

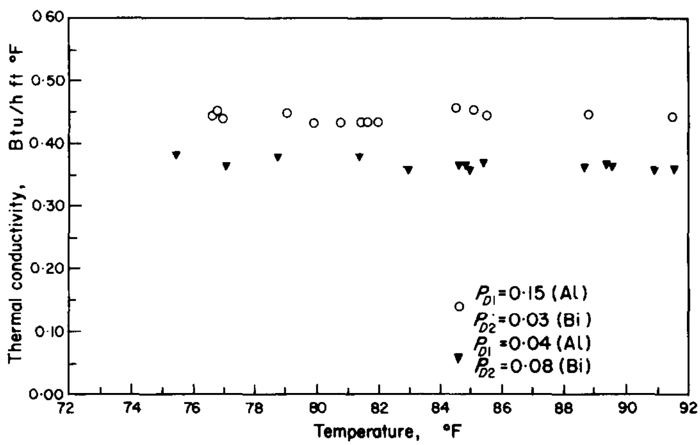

FIG. 3. Thermal conductivity vs. temperature for rubber–aluminum–bismuth mixtures.

discontinuous phase is small and the ratio of k_d and k_c is less than 100, such as bismuth–rubber mixtures, Maxwell's equation predicts the thermal conductivity with a reasonable accuracy regardless of the particle shape. Maxwell's equation was derived assuming spherical particles as the discontinuous phase.

4. In two-phase mixtures, if the ratio of k_d and k_c is greater than 100, such as aluminum–rubber mixtures, the particle shape of the discontinuous phase will affect the thermal conductivity of mixtures. However, since the shape of aluminum particles is irregular, Cheng's equation gives a better prediction than Maxwell's equation. This is because Cheng's equation was derived by assuming the mixture contained all kinds of different particle shapes.

5. The comparison of the predicted thermal conductivity of nickel–rubber mixtures with experimental data shows that Maxwell's equation gives the better results. This can be explained from the fact that the nickel particles are almost spherical in shape.

6. The comparison of the theoretically predicted thermal conductivity of three-phase mixtures with experimental data shows that Cheng's analysis always over-estimated (+12·05 per cent) while Hamilton's equation always under-estimated (−13·3 per cent). The values from these two techniques can be served as the upper and lower bounds for the theoretical prediction of the thermal conductivity of three-phase mixtures. Further study in three-phase mixtures is required.

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